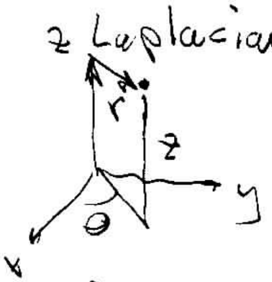
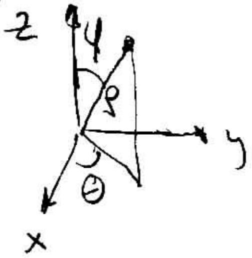



Laplacian in cart $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

in cyl $\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(rT\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}$




in sph $\nabla^2 T = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial T}{\partial \rho}\right) + \frac{1}{\rho^2 \sin^2\varphi}\frac{\partial^2 T}{\partial \theta^2}$

$+ \frac{1}{\rho^2 \sin\varphi}\frac{\partial}{\partial \varphi}\left(\sin\varphi \frac{\partial T}{\partial \varphi}\right)$

"radial" part of $\nabla^2 T$

Recall we were solving $\boxed{\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T}$ in Cart, Cyl, Sph.

or

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r}\frac{\partial}{\partial r}\left(rT\right)$$

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2 \frac{\partial T}{\partial \rho}\right)$$ (we might use $r$ instead of $\rho$!)

I.C. $\theta\big|_{t=0} = \theta_i$

B.C. $\frac{\partial \theta}{\partial n}\Big|_{n=0} = 0$ (or $T$ is finite at "center" $n=0$)

and at $n \rightarrow$ solid boundary (in "radial" direction) $\boxed{n=1}$

$$\frac{\partial \theta}{\partial n}\Big|_{n=1} = -Bi\,\theta\Big|_{n=1}$$

$\theta = \frac{T - T_\infty}{T_i - T_\infty}$ ← fluid Temp

The solutions for $\theta = \frac{T - T_\infty}{T_i - T_\infty}$ are

Cart, wall $\left(\frac{L}{2}\right)$

$$\theta = \sum_{n=1}^{\infty} \underbrace{\left[\frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}\right]}_{A_n} e^{-\lambda_n^2 \tau} \cos\left(\frac{\lambda_n x}{L}\right)$$

where $\lambda_n \tan \lambda_n = Bi$

Cyl

R = cyl radius

$$\theta = \sum_{n=1}^{\infty} \underbrace{\left(\frac{2}{\lambda_n}\right)\left(\frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)}\right)}_{A_n} e^{-\lambda_n^2 \tau} J_0\left(\lambda_n \frac{r}{R}\right)$$

where $\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = Bi = \frac{hR}{k}$

Sph

R = sph. radius

$$\theta = \sum_{n=1}^{\infty} \underbrace{\left[\frac{4(\sin(\lambda_n) - \lambda_n \cos(\lambda_n))}{2\lambda_n - \sin(2\lambda_n)}\right]}_{A_n} e^{-\lambda_n^2 \tau} \frac{\sin\left(\lambda_n \frac{\rho}{R}\right)}{\left(\lambda_n \rho/R\right)}$$

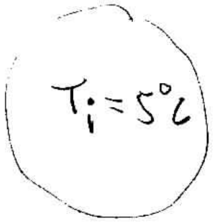
where $1 - \lambda_n \cot(\lambda_n) = Bi = \frac{hR}{k}$

If $\tau > 0.2$

- Drop $\sum_{n=1}^{\infty}$ and set $n=1$ to use 1-term approx.
- $A_1$ and $\lambda_1$ values from Tables!
- $J_0(\sim)$ + $J_1(\sim)$ from Tables

//

Ex| How we boil an egg $2R = D = 5\,cm$ or $R = 0.025\,m$




$T_\infty = 95°C$

$h = 1200 \frac{W}{m^2 K}$

How long to reach $T = 70°C$ at center of egg.

Egg is 74% $H_2O$ (mostly $H_2O$)

use $k\Big|_{H_2O \text{ @ avg } T = \frac{(5+70)°C}{2}} = 0.627 \frac{W}{mK}$

$$\alpha = \frac{k}{\rho C_p} = 0.151 \times 10^{-6} \frac{m^2}{s}$$

Geek info: white thickens @ 63°C

hardens @ 65°C

yolk thickens @ 65°C

hardens @ 70°C

whole egg hardens above 70°C

Here we go...

$$Bi = \frac{hR}{k} = \ldots = 47.8 > 0.1 \Rightarrow \left(\text{can use 1-term approx.?}\right)$$

<u>Can't</u> use lumped analysis!!

From a Table $\lambda_1 = 3.0754 \quad A_1 = 1.9958$

So $\frac{T|_{\xi=0} - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$

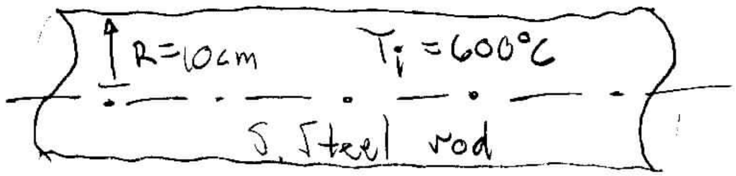
$$\frac{(70-95)°C}{(5-95)°C} = A_1 e^{-\lambda_1^2 \tau} \Rightarrow \tau = 0.209 > 0.2 \Rightarrow \text{ok to use 1-term approx.}$$

But $\tau = \dfrac{\alpha t}{R^2}$ or $t = \dfrac{\tau R^2}{\alpha} = 865\,s \approx 14.4$ min

Call it 15 min

Hey, at 5280+ feet, at what T does $H_2O$ boil?

Ex| cooling chamber $T_\infty = 200°C$ $\quad h = 80\ W/m^2K$




What is $T_{c_L}$ after 45 min of cooling?

What was the heat transfer per unit length during cooling?

Material prop. of S.S. at room temp: $k = 14.9\ W/mK$ $\quad \rho = 7900\ kg/m^3$ $\quad c_p = 477\ \dfrac{J}{kg\,K}$

$\alpha = 3.95 \times 10^{-6}\ m^2/s$

(could also use mtl. prop. @ $T_{average}$)

$Bi = \dfrac{hR}{k} = 0.5369$ $\qquad \tau = \dfrac{\alpha t}{R^2} = \ldots = 1.066$

Look in Table for cyl. soln... $\lambda_1 = 0.970$ $\quad A_1 = 1.122$

want this

So $\Theta_{c_L} = \dfrac{T_{c_L} - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = 0.412$

so $T_{c_L} = 200°C + (600 - 200)°C \times 0.412 = 364°C$

at $t = 45$ min

So $\dfrac{Q}{Q_{\substack{max \\ possible}}} = 1 - 2\theta_t \dfrac{J_1(\lambda_1)}{\lambda_1} = 1 - 2 \times 0.412\left(\dfrac{0.430}{0.970}\right) = 0.636$

Note $\lambda_1 = 0.970$ from Table

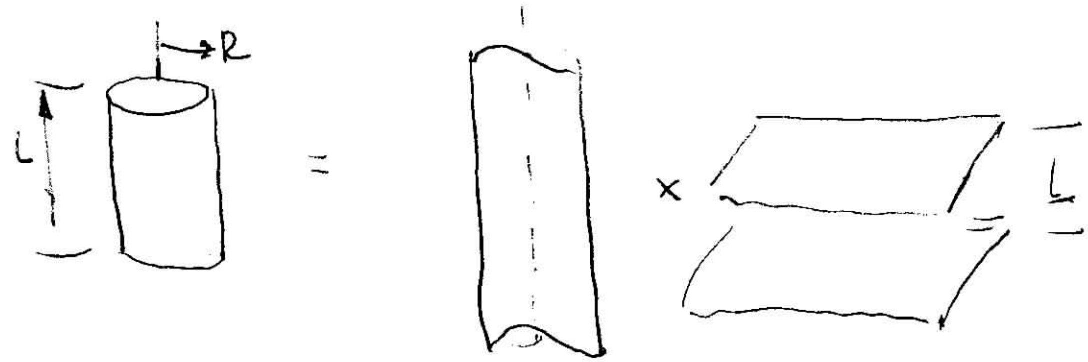
$J_1(\lambda_1) = 0.430$ Table value

But $Q_{\substack{max \\ possible}} = m c_p (T_i - T_\infty) = \rho(\pi R^2 L) \cdot c_p (600-200)°C$

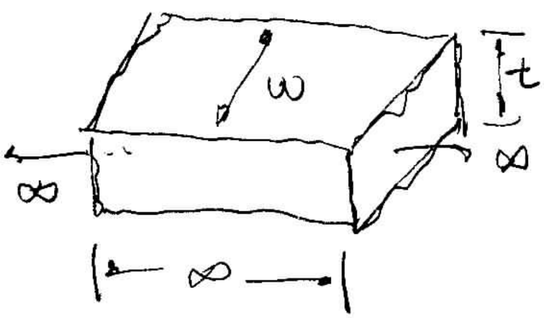
$= 47{,}350 \text{ kJ}$

Thus $Q = 0.636\, Q_{\substack{max \\ pos.}} = 30{,}120 \text{ kJ}$

Multi-D objects (jtmot method)




$$\left(\frac{T(t,x,r)-T_\infty}{T_i - T_\infty}\right)_{\substack{\text{short}\\ \text{cyl}}} = \left(\frac{T(t,x)-T_\infty}{T_i - T_\infty}\right)_{\substack{\text{flat}\\ \text{plate}}} \times \left(\frac{T(t,r)-T_\infty}{T_i - T_\infty}\right)_{\substack{\infty\text{ long}\\ \text{cyl}}}$$




$$\left(\frac{T(t,x,y)-T_\infty}{T_i - T_\infty}\right)_{\substack{\infty\text{ long}\\ \text{rectangular}\\ \text{bar}}} = \left(\frac{T(t,x)-T_\infty}{T_i - T_\infty}\right)_{\substack{\text{horiz.}\\ \parallel\text{ plates}\\ \text{wall}}} \times \left(\frac{T(t,y)-T_\infty}{T_i - T_\infty}\right)_{\substack{\text{vert.}\\ \parallel\text{ plate}\\ \text{wall}}}$$